**8.1 Finding the Domains of Rational Functions and Simplifying Rational Expressions**

**Objective 1:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* A **rational function** is a function whose equation can be put into the form

where \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* + Example of a rational function:

Example: Evaluate the function $f\left(x\right)= \frac{x-5}{x^{2}-4}$ at the indicated values.

1. $f\left(6\right)$
2. $f(-3)$

**Objective 2:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* Consider the function $\left(x\right)=\frac{5}{x}$ . What happens when x = 0? Graph the function on your calculator and sketch it below. What do you notice?
* A **vertical asymptote** is a vertical line that corresponds to the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
	+ The graph will \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ intersect the asymptote and is not part of the graph of a function.
* The **domain** of a rational function is the set of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ except for any numbers which, when substituted for $x$, would make the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
	+ These values are called the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example: Find the domain of the rational function. Identify any vertical asymptotes.

1. $f\left(x\right)=\frac{x-4}{x^{2}+5x+2}$

1. $g\left(x\right)=\frac{x^{2}-5x+1}{x^{2}+10}$

**Objective 3:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* To simplify a rational expression: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example: Simplify the expression. State the domain of the function.

1. $f\left(x\right)= \frac{2x^{2}-6x-20}{2x^{2}-50}$
2. $f\left(x\right)= \frac{x^{3}-5x^{2}+6x}{x^{3}-2x^{2}-9x+18}$
3. $f\left(x\right)= \frac{x^{2}-x-12}{4-x}$
4. $f\left(x\right)= \frac{x^{2}-8xy+16y^{2}}{x^{2}-16y^{2}}$

**Objective 4:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* A **quotient function** of $f$ and $g$: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example: Let $f\left(x\right)=8x^{3}-125$ and $\left(x\right)=4x^{2}-25$ .

1. Find an equation of $\frac{f}{g}$. Simplify the right-hand side of the equation.
2. Find $\left(\frac{f}{g}\right)\left(3\right).$

**Objective 5:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* A **rational model** is a ­­­­­­­­­­­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, or its graph, that describes an \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
	+ Percentage Formula: If m out of n items have a certain attribute, then the percentage $p$ of the n items that have the attribute is:

Example: A reasonable model of the number of Internet users in the United States is $I\left(t\right)=20.72t-83.94$, where $I(t)$ is the number of Internet users (in millions) at $t$ years since 1990. A reasonable model of the US population is $U\left(t\right)=0.0068t^{2}+2.58t+251.9$, where $U\left(t\right)$ is the US population (in millions) at $t$ years since 1990.

1. Let $P(t)$ be the percentage of Americans who are Internet users at $t$ years since 1990. Find an equation of $P$.
2. Use $P$ to estimate the percentage of Americans who were Internet users in 2004. Then compute the actual percentage using the 2004 number of internet users as 207 million and the US population as 293.7 million. Is your result from using the model an underestimate or an overestimate?
3. Find $P(17)$. What does it mean in this situation?

**8.2 Multiplying and Dividing Rational Expressions**

**Objective 1:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* To multiply rational expressions $\frac{A}{B} and\frac{C}{D}$:

Example: Find the product of the following rational expressions. Simplify the result.

1. $\frac{4x^{3}}{7x-1} ·\frac{3x+5}{2x}$
2. $\frac{k^{2}-9}{2k^{2}-k-10} ·\frac{4k^{2}-25}{k^{2}+4k-21}$

Example: Let $f\left(x\right)=\frac{35x^{2}-25x}{x^{2}-36}$ and $g\left(x\right)=\frac{6-x}{15x^{4}}$.

1. Find an equation of the product function.
2. Find $(f·g)(2)$.

**Objective 2:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* To divide rational expressions $\frac{A}{B} and\frac{C}{D}$:

Example: Find the quotient of the following rational expressions.

1. $\frac{6x^{2}}{x-3} ÷\frac{4x^{7}}{x+1}$

1. $\frac{x^{3}-y^{3}}{9x^{2}-y^{2}} ÷\frac{x^{3}+x^{2}y+xy^{2}}{6x^{2}-xy-y^{2}}$

Example: Let $f\left(x\right)=\frac{81x^{2}-49}{3x^{2}+16x+5}$ and $g\left(x\right)=\frac{7-9x}{18x+6}$.

1. Find an equation of the quotient function.
2. Find $\left(\frac{f}{g}\right)(3)$.

Example: Perform the indicated operations.

1. $\left(\frac{x^{2}-2x-48}{x^{2}+8x+16} ÷\frac{3x^{2}-9x}{x^{2}-16}\right) ·\frac{6x+24}{5x+30}$

**8.3 Adding and Subtracting Rational Expressions**

**Objective 1:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* To add rational expressions $\frac{A}{B} and\frac{C}{B}$:

Example: Find the sum of the rational expressions. Simplify if possible.

1. $\frac{x^{2}+5x}{x^{2}-9}+\frac{6}{x^{2}-9}$
2. $\frac{5}{6x}+\frac{9}{4x^{3}}$
3. $\frac{2}{p+3}+\frac{4}{p-5}$
4. $\frac{3x}{x^{2}+2xy+y^{2}}+\frac{2y}{x^{2}-y^{2}}$

Example: Let $f\left(x\right)=\frac{3}{12x^{3}-22x^{2}+6x}$ and $g\left(x\right)=\frac{x+1}{30x^{2}-10x}$.

1. Find an equation of the sum function.
2. Find $(f+g)(2)$.

**Objective 2:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* To subtract rational expressions $\frac{A}{B} and\frac{C}{B}$:

Example: Find the difference of the following rational expressions.

1. $\frac{x^{2}}{x+1}- \frac{x+2}{x+1}$

1. $\frac{5}{4ab^{2}}-\frac{3}{2a^{3}b}$
2. $\frac{3x-1}{2x^{2}-7x-4}-\frac{5}{x^{2}-8x+16} $
3. $\frac{y}{y-2}-\frac{3}{2-y}$
4. $\left(\frac{x+2}{x^{2}-x}-\frac{6}{x^{2}-1}\right)+\frac{3}{x^{2}+x}$

Example: Let $f\left(x\right)=\frac{x-1}{x+1}$ and $g\left(x\right)=\frac{x+1}{x-1}$.

1. Find an equation of the difference function.
2. Find $\left(f-g\right)(5)$.

**8.4 Simplifying Complex Rational Expressions**

**Objective 1:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* A **complex rational expression** is a rational expression whose numerator or denominator (or both) is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
	+ Example:

**Objective 2:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* To simplify complex rational expressions: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example: Simplify the complex rational expression.

1. $\frac{ \frac{12}{x} }{\frac{8}{x^{3}}}$
2. $\frac{ \frac{x^{2}-9}{x^{2}+2x+1} }{\frac{2x-6}{4x+4}}$
3. $\frac{ \frac{1}{y^{2}} + \frac{3}{2x} }{\frac{2}{y} -\frac{1}{ 3x}}$

Example: Let $f\left(x\right)=2-\frac{5}{x+2}$ and $g\left(x\right)=\frac{x}{x+2}+\frac{x+1}{x^{2}-4x-12}$.

1. Find an equation of the quotient function.
2. Find $\left(\frac{f}{g}\right)(2)$.

**8.5 Solving Rational Equations**

**Objective 1:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* A **rational equation in one variable** is an equation in one variable in which both sides can be written as \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
	+ Example:
* An  **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**  is a value that is obtained from solving for $x$ but is an excluded value for one or more of the fractions in the equation.
	+ When solving rational equations, you must \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* To solve a rational equation:
	+ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
	+ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
	+ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
	+ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example: Solve the equation. Check for extraneous solutions.

1. $\frac{2}{p}+5=\frac{8}{p}-1$
2. $2-\frac{1}{x-2}=\frac{x-3}{x-2} $
3. $\frac{x}{x+2}-\frac{7}{5-x}=\frac{14}{x^{2}-3x-10} $

Example: Let $f\left(x\right)=\frac{x+1}{x-3}-\frac{x-2}{x+3} .$ Find $x$ when $f\left(x\right)=1.$

**Objective 2:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example: The following formula is useful in electronics: $I=\frac{ε}{R+r}$. Here, $I$ is the current in an electrical circuit, $ε$ is the electromotive force, $R$ is the circuit’s resistance, and $r$ is the battery’s resistance. Solve the formula for the variable $R$.

**Objective 3:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example: In example 9 of section 8.1, we found the model $\left(t\right)=\frac{2072t-8394}{0.0068t^{2}+2.58t+251.9}$ , where $P(t)$ is the percentage of Americans who are Internet users at $t$ years since 1990. Estimate when 85% of Americans were Internet users and predict when 85% of Americans will be Internet users.

**8.6 Modeling with Rational Functions**

**Objective 1:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* **Computing the mean:** If a quantity $Q$ is divided into $n$ parts, the mean amount $M$ of the quantity per part is given by

Example: The underground band Melted Zipper wants to make and sell a CD of its original songs. It costs about $1000 to record the music onto a digital audiotape (DAT), $100 to rearrange the music and improve the sound quality, $350 for artwork for the cover and inside leaflet and $350 to set up production. In addition, it will cost $2.50 for each CD manufactured.

1. What is the total cost of making 300 CDs?
2. Let $C\left(n\right)$ be the total cost (in dollars) of making $n$ CDs. Find an equation of $C$.
3. Let $P(n)$ be the price (in dollars) the band should set for each CD so that it breaks even by making and selling $n$ CDs. Find an equation of $P$.
4. Find $P(300)$. What does it mean in this situation?
5. Find $n$ when $P\left(n\right)=10$. What does it mean in this situation?

Example: In Example 10 of Section 7.5, we modeled the annual US consumption of bottled water. A reasonable model is $B\left(t\right)=21t^{2}+42t+2247$, where $B(t)$ is US bottled water consumption ( in millions of gallons) in the year that is $t$ years since 1990. In Exercise 9 of Homework 7.7, you modeled the US population. A reasonable model is $U\left(t\right)=0.0068t^{2}+2.58t+251.9$, where $U\left(t\right)$ is the US population (in millions) at $t$ years since 1990.

1. Let $M(t)$ be the annual mean consumption of bottled water per person (in gallons per person) in the year that is $t$ years since 1990. Find an equation of $M$.
2. Find $M(24)$. What does it mean in this situation?
3. Find $t$ when $M\left(t\right)=51$. What does it mean in this situation?

**Objective 2:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |
| --- | --- | --- |
| Year | Broadband CableSubscribers | DSL Subscribers |
| 2002 | 5.4 | 10.7 |
| 2003 | 8.9 | 13.4 |
| 2004 | 12.6 | 17.0 |
| 2005 | 15.1 | 19.7 |
| 2006 | 17.3 | 22.0 |
| 2007 | 19.5 | 23.7 |
| 2008 | 21.7 | 25.0 |

Example: The numbers of broadband cable subscribers and DSL subscribers in the United States are shown in the tale for various years. Let $B(t)$ be the number of broadband cable subscribers and $D(t)$ be the number of DSL subscribers, both in millions, at $t$ years since 1990.

1. Find an equation of $B$ and $D$.
2. Find an equation of the sum function $B+D$. What do the inputs and outputs of $B+D$ mean in this situation?
3. In exercise 30 of Homework 3.1, you modeled the number of US households. A reasonable model is $H\left(t\right)=1.4t+91.7$, where $H(t)$ is the total number (in millions) of US households at $t$ years since 1990. Let $P(t)$ be the percentage of US households that are broadband cable or DSL subscribers. Find an equation of $P$. Assume that no household subscribes to both services.
4. Find $t$ when $P\left(t\right)=38$. What does it mean in this situation?

**Objective 3:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* If an object is moving at a constant speed $s$ for an amount of time $t$, then the distance $d$ traveled is given by

Example: A person plans to drive a steady 55 mph on an 80-mile trip. Compute the driving time.

Example: A student at Seattle Central Community College plans to drive from Seattle, Washington, to Eugene, Oregon. The speed limit is 70 mph in Washington and 65 mph in Oregon. She will drive 164 miles in Washington, then 121 miles in Oregon.

1. If the student drives steadily at the speed limits, compute the driving time.
2. If the student exceeds the speed limits, let $T(a)$ be the driving time (in hours) at $a$ mph above the speed limits. Find an equation of $T$.
3. Find $T(0)$. Compare this result with the result in Problem 1.
4. If the student drives 5 mph over the speed limits, compute the driving time.
5. If the student wants the driving time to be 4 hours, how much over the speed limits would she have to drive?

**8.7 Variation**

**Objective 1:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* If $y = kx$ for some nonzero constant $k$, we say that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ or that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
	+ $k $is called the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ or the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
	+ The equation $y=kx$ is called a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* If $y=\frac{k}{x} $for some nonzero constant $k$, we say that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ or that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
	+ The equation $y=\frac{k}{x}$ is called an \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Objective 2:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* The graph of a direct variation equation is a line with \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* In a direct variation equation:
	+ If the value of $x$ increases, then the value of $y$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
	+ If the value of $x$ decreases, then the value of $y$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* In an inverse variation equation:
	+ If the value of $x$ increases, then the value of $y$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
	+ If the value of $x$ decreases, then the value of $y$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Objective 3:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* To find a direct variation equation: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* To find an inverse variation equation: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example: The variable $y$ varies directly as $x$ with positive variation on constant $k$.

1. What happens to the value of $y$ as the value of $x$ increases?
2. If $y=5$ when $x=3$, find an equation for $x$ and $y$.

Example: The variable $y$ varies inversely as $x$ with the positive variation constant $k$.

1. For positive values of $x$, what happens to the value of $y$ as the value of $x$ increases?
2. If $y=4$ when $x=2$, find an equation for $x$ and $y$.

**Objective 4:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example: Assume that the bounce height $B$ (in inches) of a tennis ball varies directly as the drop height $d$ (in inches). The bounce height of the tennis ball is 20 inches when the ball is dropped from an initial height of 30 inches.

1. Find an equation of $B$ and $d$.
2. Estimate the bounce height if the drop height is 50 inches.

|  |  |
| --- | --- |
| Volume (cm3) | Pressure (atm) |
| 3 | 2.23 |
| 4 | 1.76 |
| 5 | 1.46 |
| 6 | 1.23 |
| 7 | 1.05 |
| 8 | 0.93 |
| 9 | 0.83 |
| 10 | 0.74 |
| 11 | 0.67 |
| 12 | 0.60 |
| 13 | 0.56 |
| 14 | 0.52 |
| 15 | 0.48 |
| 16 | 0.44 |
| 17 | 0.42 |
| 18 | 0.39 |
| 19 | 0.37 |
| 20 | 0.35 |

Example: The more you squeeze a sealed syringe filled with air, the harder it gets to squeeze it further. Some air volumes in cubic centimeters (cm3) and corresponding pressures in atmospheres (atm) in a sealed syringe are given in the table. Let $P$ be the pressure (in atm) in the syringe at air volume $V$ (in cm3).

1. As the value of $V$ increases, what happens to the value of $P$, according to the model? What does that pattern mean in this situation?
2. Find an equation for $V$ and $P$.
3. Estimate at what volume the pressure will be 5 atm.