Algebra 2

Name: _____

Semester 2 STUDY GUIDE

The exam is worth 20% of your final grade. On the exam, you will be permitted to use one 3" by 5" note card with your notes on it. The notes on the notecard must be hand written by you in your own handwriting.

On the exam, show your work whenever possible and circle your answer as appropriate.

1. Chapter 6 – Polynomial Functions
   a. Let $f(x) = 4x^2 + 5x - 9$, $g(x) = 6x^2 - 3x + 7$ and $h(x) = 2x - 4$.
      i. Find $f + g$. $(4x^2+5x-9)+(6x^2-3x+7)$
      $10x^2+2x-2$

      ii. Find $f - g$. $(4x^2+5x-9)-(6x^2-3x+7)$
      $-2x^2+8x-16$

      iii. Find $f \cdot g$. $(4x^2+5x-9)(6x^2-3x+7)$
      $24x^4-12x^3+28x^2+30x^3-15x^2+35x-54x^2+27x-63$
      $24x^4+18x^3-41x^2+62x-63$

      iv. Find h^2. $(2x-4)^2=(2x-4)(2x-4)$
      $4x^2-8x-8x+16$
      $4x^2-16x+16$

   b. Factor the following expressions.
      i. $5x^3 + 40$ $=5(x^3+8)$ ← sum of cubes
      $5(x+2)(x^2-2x+4)$

      ii. $81x^4 - 1$ $=(9x^2+1)(9x^2-1)$ ← diff of squares
      $(9x^2+1)(3x+1)(3x-1)$

c. Solve the following equations by factoring.

i. $4x^2 - 8x = 32$ → $4x^2 - 8x - 32 = 0$

$4(x^2 - 2x - 8) = 0$

$4(x-4)(x+2) = 0$

$x - 4 = 0$ → $x = 4$ $\quad$ $x + 2 = 0$ → $x = -2$

ii. $9x^3 - 12 = 4x - 27x^2$

factor by grouping → $9x^3 + 27x^2 - 4x - 12 = 0$

$9x^2(x+3) - 4(x+3) = 0$

$(9x^2 - 4)(x+3) = 0$

$(3x+2)(3x-2)(x+3) = 0$

$3x + 2 = 0$ → $x = -2/3$ $\quad$ $3x - 2 = 0$ → $x = 2/3$ $\quad$ $x + 3 = 0$ → $x = -3$

iii. $3r^2 = 6r$ → $3r^2 - 6r = 0$

$3r(r-2) = 0$

$3r = 0$ → $r = 0$ $\quad$ $r - 2 = 0$ → $r = 2$

iv. $16x^2 = 25$ → $16x^2 - 25 = 0$

$(4x+5)(4x-5) = 0$

$4x + 5 = 0$ → $x = -5/4$ $\quad$ $4x - 5 = 0$ → $x = 5/4$

2. Chapter 7 – Quadratic Functions

a. Solve the following equations using the square root property.

i. $3p^2 - 11 = 3$

$3p^2 = 14$

$p^2 = 14/3$ $\quad$ isolate "p^2"

$p = \pm\sqrt{14/3}$ $\quad$ square root both sides

$p = \pm\frac{\sqrt{42}}{3}$ $\quad$ rationalize

ii. $5(x-6)^2 + 3 = 33$

$5(x-6)^2 = 30$

$(x-6)^2 = 6$

$x - 6 = \pm\sqrt{6}$

$x = 6 \pm \sqrt{6}$

b. Solve the following equations using the quadratic formula.

i. $x^2 = -2x + 5$ → $x^2 + 2x - 5 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 + 20}}{2}$$

$$x = \frac{-2 \pm \sqrt{24}}{2}$$

reduce the 2s

$$x = \frac{-2 \pm 2\sqrt{6}}{2}$$

$$\boxed{x = -1 \pm \sqrt{6}}$$

ii. $2x^2 = 5x + 4$ → $2x^2 - 5x - 4 = 0$

$$x = \frac{5 \pm \sqrt{25 - 4(2)(-4)}}{4}$$

$$\boxed{x = \frac{5 \pm \sqrt{57}}{4}}$$

c. Solve the following equations by completing the square.

i. $8x^2 + 4x - 3 = 0$

$$8x^2 + 4x = 3$$

$$x^2 + \frac{1}{2}x = \frac{3}{8}$$

$$x^2 + \frac{1}{2}x + \frac{1}{16} = \frac{3}{8} + \frac{1}{16}$$

$$\left(x + \frac{1}{4}\right)^2 = \frac{7}{16}$$

$$x + \frac{1}{4} = \pm\frac{\sqrt{7}}{4}$$

$$\boxed{x = \frac{-1 \pm \sqrt{7}}{4}}$$

ii. $x^2 + 5x + 7 = 0$

$$x^2 + 5x = -7$$

$$x^2 + 5x + \frac{25}{4} = -7 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = -\frac{3}{4}$$

$$x + \frac{5}{2} = \pm\frac{i\sqrt{3}}{2}$$

$$\boxed{x = \frac{-5 \pm i\sqrt{3}}{2}}$$

d. Find an equation of the parabola that contains the points $(1, 6), (2, 11)$, and $(3, 18)$.

$6 = a(1)^2 + b(1) + C$ → $6 = a + b + C$

$11 = a(2)^2 + b(2) + C$ → $11 = 4a + 2b + C$

$18 = a(3)^2 + b(3) + C$ → $18 = 9a + 3b + C$

$5 = 3a + b$

$12 = 8a + 2b$

$10 = 6a + 2b$

$12 = 8a + 2b$

$2 = 2a$

$a = 1$

$5 = 3(1) + b$

$b = 2$

$6 = 1 + 2 + C$

$C = 3$

$$\boxed{Y = x^2 + 2x + 3}$$

e. The percentages of California's population who are foreign born and the percentages who were born in other U.S. states are listed in the table for various years.

i. Let $f(t)$ and $g(t)$ be the percentages of California's population that are foreign born and born in other U.S. states, respectively, at t years since ~~1990~~ 1900. Find and verify regression equations of f and g.

$f(t) = .0111t^2 - 1.3199t + 48.2399$

$g(t) = -.0117t^2 + 1.1660t + 22.7863$

Year	Percent Foreign Born	Percent Born in other U.S. States
1930	18.9	47.0
1940	13.4	50.0
1950	10.0	53.0
1960	8.5	51.0
1970	8.8	47.9
1980	15.1	39.5
1990	21.7	31.8
2000	25.9	23.5

ii. Estimate when the percentages of foreign born and those born in other U.S. states were equal. What is that percentage?

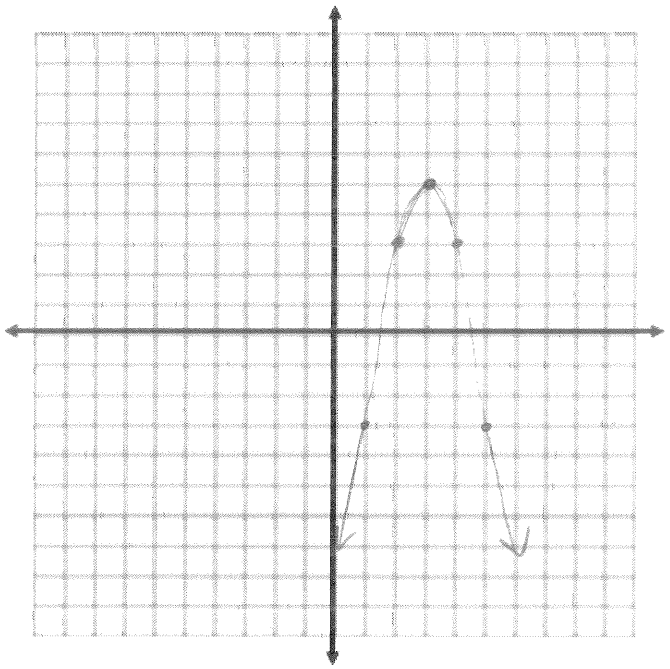
(11, 35) and (98, 25)

1911, ~35%

1998, ~25%

f. Graph each function by hand. Identify the vertex, domain, and range.

i. $y = -2(x-3)^2 + 5$



Vertex: (3, 5)

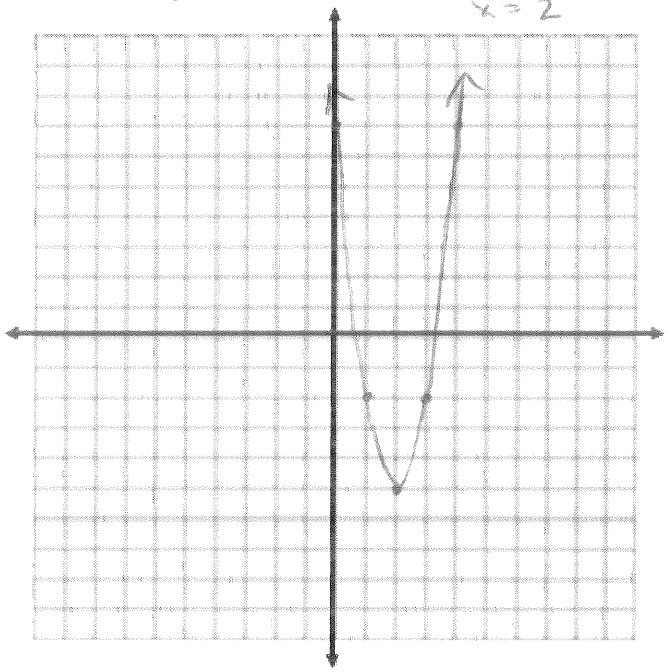
Domain: $\mathbb{R}$

Range: $y \leq 5$

ii. $y = 3x^2 - 12x + 7$

$x = \frac{-b}{2a}$

$x = 2$



Vertex: (2, -5)

Domain: $\mathbb{R}$

Range: $y \geq 5$

3. Chapter 8 – Rational Functions

a. Find the domain of the function $f(x) = \frac{5}{6x^2+11x-10}$.

$6x^2+11x-10=0$

$(3x-2)(2x+5)=0$

$3x-2=0$ $\quad x=2/3$ $\qquad 2x+5=0$ $\quad x=-5/2$

b. Perform the indicated operation.

i. $\frac{5x^4}{3x^2+6x+12} \cdot \frac{x^3-8}{15x^7}$ $\quad (x-2)(x^2+2x+4)$

$3(x^2+2x+4)$ $\qquad 3 \quad x^3$

$$\frac{(x-2)}{9x^3}$$

ii. $\frac{p^2-4t^2}{p^2+6pt+9t^2} \div \frac{p^2-3pt+2t^2}{p^2+3pt}$ $\quad \frac{(p-2t)(p+2t)}{(p+3t)(p+3t)} \cdot \frac{p(p+3t)}{(p-2t)(p-t)}$

$$\frac{p(p+2t)}{(p+3t)(p-t)}$$

iii. $\frac{5x+12}{-2x^2-8x} - \frac{2x+1}{x^2+2x-8}$ $\quad -2x(x+4)$ $\quad (x+4)(x-2)$ $\quad -2x$

$\frac{x-2}{x-2}$

$\frac{(x-2)(5x+12)}{-2x(x-2)(x+4)} - \frac{-2x(2x+1)}{-2x(x-2)(x+4)} = \frac{5x^2+2x-24-(-4x^2-2x)}{-2x(x-2)(x+4)}$

$$= \frac{9x^2+4x-24}{-2x(x-2)(x+4)}$$

iv. $\frac{x+2}{x^2-9} + \frac{3}{x^2+11x+24}$ $\quad (x-3)(x+3)$ $\quad (x+8)(x+3)$ $\quad x-3$

$\frac{x+8}{x+8}$ $\qquad x-3$

$\frac{(x+8)(x+2)}{(x+8)(x-3)(x+3)} + \frac{3(x-3)}{(x+8)(x-3)(x+3)} = \frac{x^2+10x+16+3x-9}{(x+8)(x-3)(x+3)}$

$$= \frac{x^2+13x+7}{(x+8)(x-3)(x+3)}$$

c. Simplify the complex fraction $\dfrac{\frac{x}{x-4}-\frac{2x}{x+1}}{\frac{x}{x+1}-\frac{2x}{x-4}}$.

$$\frac{\frac{x(x+1)-2x(x-4)}{(x+1)(x-4)}}{\frac{x(x-4)-2x(x+1)}{(x+1)(x-4)}} = \frac{x^2+x-2x^2+8x}{(x+1)(x-4)} \times \frac{(x+1)(x-4)}{x^2-4x-2x^2-2x}$$

$$= \frac{-x^2+9x}{-x^2-6x} = \frac{-x(x-9)}{-x(x+6)} = \boxed{\frac{x-9}{x+6}}$$

d. Solve the equation.

i. $\dfrac{2}{x-1}-\dfrac{5}{x+1}=\dfrac{4x}{x^2-1}$ $\quad (x+1)(x-1)$

$$2(x+1)-5(x-1)=4x$$
$$2x+2-5x+5=4x$$
$$-3x+7=4x$$
$$7=7x$$
$$x=1$$

$\boxed{\text{No Solution}}$

ii. $\dfrac{5}{x-3}=\dfrac{x}{x-2}+\dfrac{x}{x^2-5x+6}$ $\quad (x-3)(x-2)$

$x=1$ ← extraneous

$$5(x-2)=x(x-3)+x$$
$$5x-10=x^2-3x+x$$
$$0=x^2-8x+x+10$$
$$0=x^2-7x+10$$
$$0=(x-5)(x-2)$$

$x-5=0$ $\quad \boxed{x=5}$

$x-2=0$ $\quad x=2$ ← extraneous

4. Chapter 9 – Radical Functions

a. Simplify.

i. $\sqrt{24x^5y^{10}}$ $\quad 6 \quad 4 \quad x^4 \quad x$

$= \boxed{2x^2y^5\sqrt{6x}}$

ii. $\sqrt[3]{(24x^{10}y^{24})}$ $\quad 8 \quad 3 \quad x^9 \quad x$

$= \boxed{2x^3y^8\sqrt[3]{3x}}$

iii. $\sqrt[5]{(6x+11)^{27}}$

$(6x+11)^{25}(6x+11)^2$ $\quad (6x+11)^5\sqrt[5]{(6x+11)^2}$

iv. $\frac{4\sqrt[3]{x}}{6\sqrt[5]{x}} \cdot \frac{\sqrt[5]{x^4}}{\sqrt[5]{x^4}} = \frac{24\sqrt[3]{x}\sqrt[5]{x^4}}{36\sqrt[5]{x^5}} = \frac{2\sqrt[3]{x}\sqrt[5]{x^4}}{3x} = \boxed{\frac{2\sqrt[15]{x^{17}}}{3x}}$

v. $\frac{\sqrt{x}+1}{2\sqrt{x}-3} \quad \frac{2\sqrt{x}+3}{2\sqrt{x}+3} = \frac{2x+3\sqrt{x}+2\sqrt{x}+3}{4x-9} = \boxed{\frac{2x+5\sqrt{x}+3}{4x-9}}$

vi. $4\sqrt{12x^3} - 2x\sqrt{75x} + \sqrt{3x^3}$

$4 \cdot 3 \cdot x^2 \cdot x \quad 25 \cdot 3 \quad x^2 \cdot x$

$8x\sqrt{3x} - 10x\sqrt{3x} + x\sqrt{3x} = \boxed{-x\sqrt{3x}}$

vii. $(2+4\sqrt{x})(3-5\sqrt{x})$

$6 - 10\sqrt{x} + 12\sqrt{x} - 20x = \boxed{6 + 2\sqrt{x} - 20x}$

b. Solve the following equations.

i. $2\sqrt{x} + 3 = 13$

$2\sqrt{x} = 10$

$\sqrt{x} = 5$

$\boxed{x = 25}$

ii. $3\sqrt{5x-4} = 27$

$\sqrt{5x-4} = 9$

$5x - 4 = 81$

$5x = 85$

$\boxed{x = 17}$

iii. $3 - 2\sqrt{x} + \sqrt{9-x} = 0$

$(\sqrt{9-x})^2 = (2\sqrt{x} - 3)^2$

$9 - x = 4x - 12\sqrt{x} + 9$

$(-5x)^2 = (-12\sqrt{x})^2$

$25x^2 = 144x$

$25x^2 - 144x = 0$

$x(25x - 144) = 0$

$x = 0$ ↑ extraneous

$25x - 144 = 0$

$\boxed{x = \frac{144}{25}}$

c. The median heights of boys in the U.S. are listed in the table for various ages, up to 5 years. Let $h = f(t)$ be the median height (in inches) of boys who are t months of age.

Age (months)	Height (inches)
0	20.5
6	27.0
12	30.8
18	32.9
24	35.0
36	37.5
48	40.8
60	43.4

i. Find an equation of f. (0, 20.5) (48, 40.8)

$40.8 = a\sqrt{48} + 20.5$

$a = 2.93$

$y = 2.93\sqrt{x} + 20.5$

ii. Estimate the median height of boys who are 6 years old.

$y = 2.93\sqrt{72} + 20.5$

$y = 45.4$ inches

iii. Estimate the age at which the median height of boys is 3 feet.

$36 = 2.93\sqrt{x} + 20.5$

$x = 28$ months

$\sim 2\frac{1}{3}$ years old